MATH 53H - Solutions to Problem Set IV

1. You can find proofs for the claims of the Problem in the textbook of the course (Brendle, Theorem 3.1, pages 26-28).

2. Claim $\|x_{k+1} - x_k\| \leq \frac{1}{2^k}\|x_1 - x_0\|$.
Proof We use induction on k. For $k = 0$ the claim is clear. Since

$$\|x_{k+1} - x_k\| = \|F(x_k) - F(x_{k-1})\| \leq \frac{1}{2}\|x_k - x_{k-1}\|$$

the induction step is clear as well. $\square$

A standard telescoping argument together with the claim show that the sequence x_k is Cauchy and therefore converges to some $\overline{x} \in U$. Then taking the limit as $k \to \infty$ in $x_k = F(x_{k-1})$ shows that $F(\overline{x}) = \overline{x}$, as desired.

3. You can find proofs for the claims of the Problem in the textbook of the course (Brendle, Theorem 3.2, pages 28-30).

4. You can find an argument in the textbook of the course (Brendle, Theorem 3.6, Lemmas 3.7-3.9, pages 32-35).